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CHARACTERISTIC LENGTHS FOR NON-CIRCULAR DUCTS

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THE HYDRAULIC DIAMETER

IT IS engineering practice to consider the overall fully developed flow properties through non-circular conduits to be analogous to circular pipe properties. The basis for this comparison has been the hydraulic diameter on the assumption that ducts with the same area to perimeter ratio have the same bulk flow properties. In no way is the duct shape taken into account. This point is aptly illustrated in Fig. 1(a) where the diameter of the inscribed circle is the hydraulic diameter of the family of regular polygons shown. This illustration also offers an explanation for the lack of correlation of the turbulent flow of non-circular ducts with that of circular pipes.

Elaborating on this, the drag law for the smooth circular pipe is used by the implicit assumption that log law relationship holds along a radius of the duct. This assumption has been experimentally justified for a pipe. In the case of non-circular ducts it can be shown that for the correlation of the hydraulic diameter to hold, the isovels must be parallel to the boundary and that they must satisfy the log law up to the corner bisector. In actual practice, the log law is satisfied only in regions very close to the wall [1]. This is due to the geometry interacting with the turbulence to give rise to secondary mean flows [2–5]. The pattern of these secondary motions is such that, they convect higher longitudinal momentum towards the corners and by continuity considerations, along the boundaries. This leads to the deviation from the log law much earlier than in a circular pipe. Over most of the periphery, the effect of this deviation at a particular average velocity is to give a velocity profile as shown in Fig. 1(b). Hence, in the near wall region, the satisfaction of the log law is achieved by a smaller wall shear in a non-circular duct, with the reduction in the wall shear stress being dependent upon the strength of the secondary flows.

THE SUGGESTED CHARACTERISTIC LENGTH

According to Prandtl [3], the strength of the secondary flows depends on the highest and lowest curvatures of the

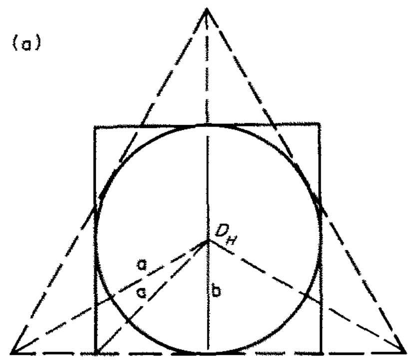

FIG. 1(a). Family of regular polygons having the same hydraulic diameter, but different suggested length scales.

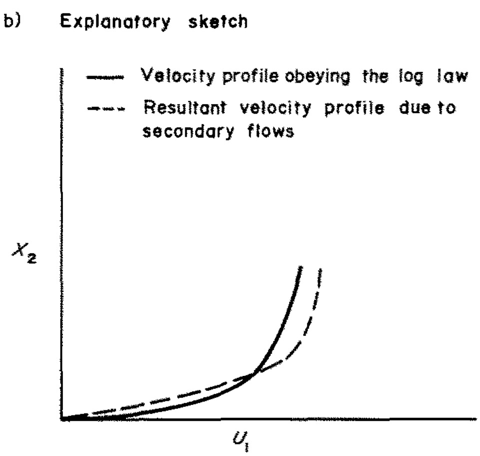

FIG. 1(b). Physical representation of actual velocity profile in a non circular duct compared to a profile satisfying the log law completely.

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isovels. Considering this aspect, a new characteristic length which takes both the highest and lowest curvature into account is suggested in Fig. 1(a). This length (D_L) is restricted to regular polygonal shapes (triangles, squares, etc.) because, as the angle between the sides reduces to less than that of an equilateral triangle, the viscous damping effects become more prominent, and will lead to large laminar regions in the corner. This would seriously violate the log law assumption even in the near wall region. The suggested characteristic length would then be too large and the obtained friction factor would be in error, as it is when calculated by using the hydraulic diameter. Finally as the number of sides of a regular polygon increase, the new characteristic length tends to the value of the hydraulic diameter, as the shape more closely approximates a circular pipe.

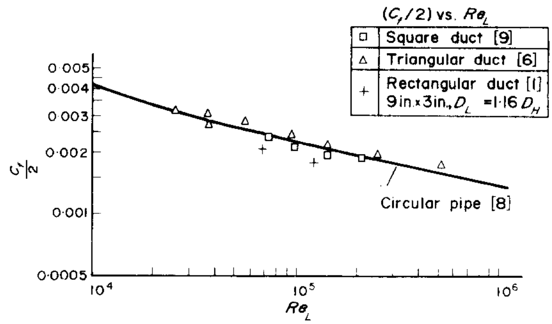

FIG. 2. Friction factors with Reynolds numbers based on the new length scale (a) square duct (b) equilateral triangular duct and (c) rectangular duct.

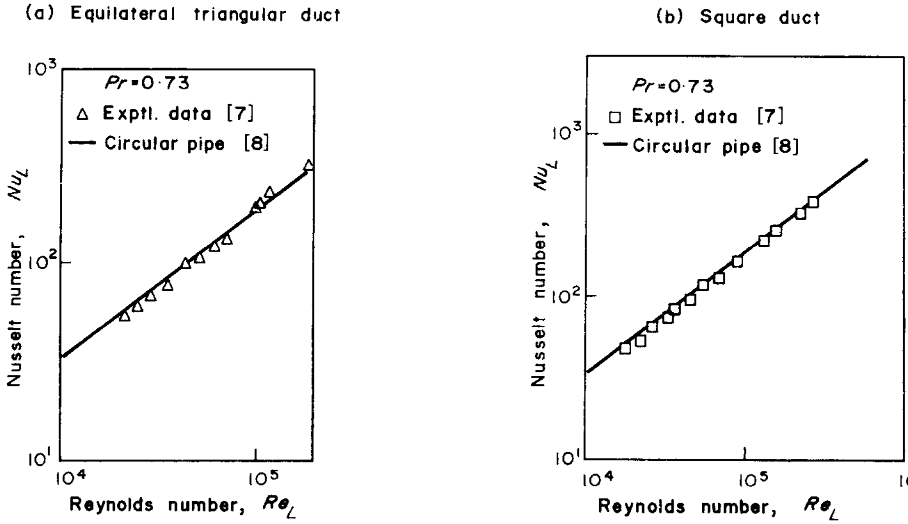

FIG. 3(a) and (b). Nusselt number plotted against Reynolds number, with the experimental data based on the new length scale.

IMPROVED CORRELATIONS

For fully developed flow, the improvement in comparison to the hydraulic diameter that is brought about in friction factor by the new characteristic length for square, equilateral triangular and rectangular ducts is seen by referring to Fig. 2. In the case of the triangular duct, the new characteristic length seems to have over-corrected the data. Curvature effects should have given stronger secondary flows, but apparently the damping effect of the sides was more effective.

For a particular average velocity, the wall shear stress for a non-circular duct is lower than for a circular pipe.

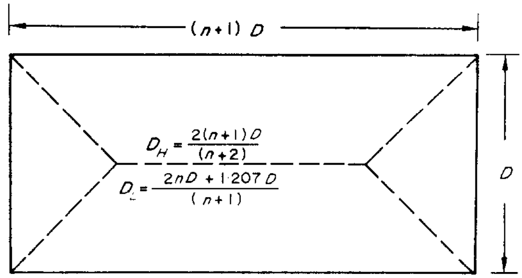

FIG. 4. New length scale for a rectangular duct.

The implication is that the velocity gradient at the wall is smaller. In practice, the heat transfer coefficients are obtained by analogy with the wall friction coefficients. By using the hydraulic diameter, the heat transfer coefficients are similarly overestimated. By incorporating the new length, the improvement in the heat transfer coefficients can be seen in the Fig. 3, for square and triangular ducts.

In the case of rectangular ducts, the secondary flows exist in strength only in regions close to the shorter sides. Brundrett and Baines [5] have shown that the corner bisector separates the secondary flow cells. Thus, the deviation from the log law is earlier in the corner region. The region of maximum influence along the longer side will be about half a shorter side length, measured from the corner. The rest of the longer side flow can be considered to be analogous to flow between parallel plates. The characteristic length scale for a rectangle should incorporate this by an appropriate weighting for the parallel plate region, as shown in Fig. 4, where $1\cdot207D$ is the value of D_L for a square duct.

Finally, for other ducts having symmetry about two axes (e.g. ellipses), it is suggested that they be approximated by straight edges. An average length scale can then be found as for a rectangle.

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ENTRANCE-REGION HEAT TRANSFER FOR LAMINAR FLOW IN POROUS TUBES

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NOMENCLATURE

B_n, series coefficients for temperature distribution;
d, tube diameter;
G_n, series coefficients for local Nusselt number;
Gz_0, entrance Graetz number, $Re_0 Pr/(x/d)$;
H_n, eigenfunction for temperature distribution;
k, thermal conductivity;
Nu, local Nusselt number; $Nu_{f.d.}$, fully developed Nusselt number;
Pr, Prandtl number;
q_w, wall heat flux due to molecular conduction;
r, radial coordinate; r_w, pipe radius;
Re_0, axial Reynolds number at $x = 0$, $\bar{u}_0 d/\nu$;
Re_w, wall Reynolds number, $v_w d/\nu$;
T, temperature; T_0, uniform entrance temperature at $x = 0$; T_w, uniform wall temperature; T_{c}, centerline temperature;
$\bar{u}_0$, mean velocity at $x = 0$;
v_w, wall injection or extraction velocity;
x, axial location measured from point where step change in wall temperature occurs.

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